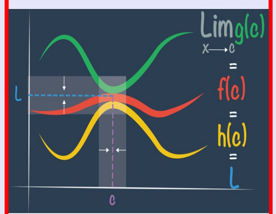
# Math 261

# Fall 2022

# Lecture 43




Feb 19-8:47 AM

1) Evaluate $\int \cos \pi x \, dx$

Let $u = \pi x$

$du = \pi \, dx$

$\frac{du}{\pi} = dx$

$$\int \cos \pi x \, dx = \int \cos u \, \frac{du}{\pi}$$

$$= \frac{1}{\pi} \int \cos u \, du = \frac{1}{\pi} \sin u + C$$

$$= \boxed{\frac{1}{\pi} \sin \pi x + C}$$

Nov 15-8:47 AM

2) $\int x^2\sqrt{x^3+1}\,dx$

Let $u = x^3+1$

$du = 3x^2dx$

$\frac{du}{3} = x^2dx$

$\int x^2\sqrt{x^3+1}\,dx = \int \sqrt{u}\,\frac{du}{3}$

$= \frac{1}{3}\int u^{1/2}du = \frac{1}{3}\cdot\frac{u^{3/2}}{3/2} + C$

$= \frac{2}{9}u\sqrt{u} + C$

$= \boxed{\frac{2}{9}(x^3+1)\sqrt{x^3+1} + C}$

Nov 15-8:49 AM

3) $\int \csc^2(4x)\,dx =$

Let $u = 4x$

$du = 4dx$

$\frac{du}{4} = dx$

$\int \csc^2(4x)dx = \int \csc^2 u\,\frac{du}{4}$

$= \frac{1}{4}\int \csc^2 u\,du = \frac{1}{4}\cdot(-\cot u) + C$

$= \boxed{\frac{-1}{4}\cot(4x) + C}$

Nov 15-8:54 AM

4) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Let $u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \frac{\sin u}{\cancel{u}} \cdot 2\cancel{u}\,du = 2\int \sin u\,du$$

$$= 2 \cdot -\cos u + C = \boxed{-2\cos\sqrt{x} + C}$$

Nov 15-8:57 AM

5) $\int \sqrt{x}\,\sec(1 + x\sqrt{x})\tan(1 + x\sqrt{x})\,dx$

$u = 1 + x\sqrt{x}$

$u = 1 + x^{3/2}$ $\quad du = \frac{3}{2}x^{1/2}dx$ $\quad \frac{2}{3}du = \sqrt{x}\,dx$

$$I = \int \sec u \tan u \cdot \frac{2}{3}du = \frac{2}{3}\int \sec u \tan u\,du$$

$$= \frac{2}{3}\sec u + C$$

$$= \boxed{\frac{2}{3}\sec(1 + x^{3/2}) + C}$$

Nov 15-9:00 AM

6) $$\int \frac{1}{\cos^2 x \cdot \sqrt{1+\tan x}}\,dx = \int \frac{\sec^2 x}{\sqrt{1+\tan x}}\,dx$$

$u = 1 + \tan x$

$du = \sec^2 x\,dx$

$$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$$

$$= \frac{u^{1/2}}{1/2} + C$$

$$= 2\sqrt{u} + C = \boxed{2\sqrt{1+\tan x} + C}$$

Nov 15-9:04 AM

7) $$\int x^2 \sqrt{2+x}\,dx$$

$u = 2 + x \rightarrow u - 2 = x$

$du = dx \qquad (u-2)^2 = x^2$

$$\int (u-2)^2 \sqrt{u}\,du =$$

$$\int (u^2 - 4u + 4)u^{1/2}\,du = \int (u^{5/2} - 4u^{3/2} + 4u^{1/2})\,du$$

$$= \frac{u^{7/2}}{7/2} - \frac{4u^{5/2}}{5/2} + 4\frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{7}(\sqrt{u})^7 - \frac{8}{5}(\sqrt{u})^5 + \frac{8}{3}(\sqrt{u})^3 + C$$

$$= \frac{2}{7}(\sqrt{2+x})^7 - \frac{8}{5}(\sqrt{2+x})^5 + \frac{8}{3}(\sqrt{2+x})^3 + C$$

$$= \frac{2}{7}(2+x)^3\sqrt{2+x} - \frac{8}{5}(2+x)^2\sqrt{2+x} + \frac{8}{3}(2+x)\sqrt{2+x} + C$$

Nov 15-9:08 AM

8) $\int x^2\sqrt{2+x}\,dx$

$u=\sqrt{2+x}$

$u^2=2+x \rightarrow x=u^2-2$

$2u\,du=dx$

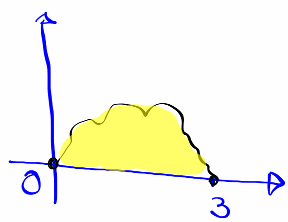
$$I=\int (u^2-2)^2\cdot u\cdot 2u\,du = 2\int (u^4-4u^2+4)u^2\,du$$

$$=2\int (u^6-4u^4+4u^2)\,du = 2\left[\frac{u^7}{7}-\frac{4u^5}{5}+\frac{4u^3}{3}\right]+C$$

$$=\frac{2}{7}\left(\sqrt{2+x}\right)^7-\frac{8}{5}\left(\sqrt{2+x}\right)^5+\frac{8}{3}\left(\sqrt{2+x}\right)^3+C$$

Nov 15-9:15 AM

9) find $\int_0^3 x\sqrt{9-x^2}\,dx$




$u=9-x^2$ $\quad x=0 \rightarrow u=9$

$du=-2x\,dx$ $\quad x=3 \rightarrow u=0$

$\frac{du}{-2}=x\,dx$ $\quad \int_0^3 x\sqrt{9-x^2}\,dx=\int_9^0 \sqrt{u}\,\frac{du}{-2}$

$$=\frac{-1}{2}\int_9^0 u^{1/2}\,du$$

Recall

$$\int_b^a f(x)\,dx=-\int_a^b f(x)\,dx$$

$$=\frac{1}{2}\int_0^9 u^{1/2}\,du=\frac{1}{\cancel{2}}\cdot\frac{u^{3/2}}{3/\cancel{2}}\Bigg|_0^9$$

$$=\frac{1}{3}u\sqrt{u}\Bigg|_0^9=\frac{1}{3}\cdot 9\sqrt{9}=\boxed{9}$$

Nov 15-9:19 AM

10) Evaluate $\int_0^1 x\sqrt{1-x^4}\,dx$

Hint: $x^4 = (x^2)^2$

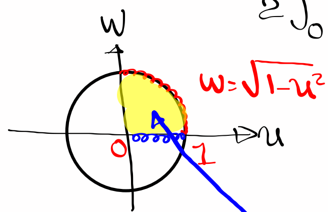
$\int_0^1 x\sqrt{1-x^4}\,dx = \int_0^1 \sqrt{1-u^2}\,\frac{du}{2}$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du$




$w = \sqrt{1-u^2}$

$w^2 = 1 - u^2$

Unit circle $\rightarrow$ $u^2 + w^2 = 1$

$\frac{1}{4}\cdot\pi r^2 = \frac{1}{4}\cdot\pi\cdot 1^2 = \frac{\pi}{4}$

Final Ans. $\frac{1}{2}\cdot\frac{\pi}{4} = \boxed{\frac{\pi}{8}}$

Nov 15-9:26 AM

11) Evaluate $\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$ using

$\int_0^\pi x f(\sin x)\,dx = \frac{\pi}{2}\int_0^\pi f(\sin x)\,dx$

$\frac{\sin x}{1+\cos^2 x} = \frac{\sin x}{1+1-\sin^2 x} = \frac{\sin x}{2-\sin^2 x}$

$f(x) = \frac{x}{2-x^2}$

$\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx = \frac{\pi}{2}\int_0^\pi \frac{\sin x}{2-\sin^2 x}\,dx = \frac{\pi}{2}\int_0^\pi \frac{\sin x}{1+\cos^2 x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= \frac{\pi}{2}\int_1^{-1} \frac{-du}{1+u^2} = \frac{\pi}{2}\int_{-1}^1 \frac{du}{1+u^2}$

$f(u) = \frac{1}{1+u^2}$

even function

$\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(x)$ is even.

$I = \frac{\pi}{\cancel{2}}\cdot\cancel{2}\int_0^1 \frac{1}{1+u^2}\,du = \pi\int_0^1 \frac{1}{1+u^2}\,du$

using table

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

$= \pi\cdot\frac{1}{1}\tan^{-1}\frac{u}{1}\Big|_0^1$

$= \pi\left[\tan^{-1}1 - \tan^{-1}0\right]$

$= \pi\left[\frac{\pi}{4} - 0\right] = \boxed{\frac{\pi^2}{4}}$

Nov 15-9:41 AM